Department of Geomatics
451-337 Satellite Positioning and Geodesy
Mid-Semester Test 2006

- WORKED SOLUTIONS -

Question 1 *(Geodetic Coordinates and Reference Frames)*

(a) Explain in detail how GDA94 has been implemented in Australia. Discuss the advantages and shortcomings of GDA94. (6 marks)

GDA94 is a realisation of ITRF92@1994.0 in Australia. This was achieved through a four stage process.

Firstly the Australian Fiducial Network (AFN) was established. The AFN consists of a number (eleven?) of permanent GPS tracking stations distributed across Australia. GPS data from this network was combined with GPS data from stations the IGS tracking network to yield 3D ITRF92@1994.0 coordinates for the AFN sites. By this means a sparse framework of points with highly accurate ITRF coordinates was established on the Australian continent.

Secondly, the Australian National Network (ANN) was observed and linked to the AFN. The ANN is a nation-wide network of non-permanent GPS stations with an average station spacing of about 500 km. These stations were occupied for an extended period time (days/weeks) and the data was combined with AFN data, then adjusted to yield ITRF92@1994.0 coordinates at the ANN sites.

Thirdly, subsidiary geodetic and survey control networks were adjusted within the AFN/ANN framework to obtain GDA94 coordinates for these existing points.

Fourthly other coordinate data (such as GIS data sets) have been (or are being) progressively moved to GDA94 via a national, non-conformal transformation model.

GDA94 provides a nationally consistent geodetic framework that is more accurate (less distorted) than any of the earlier geodetic datums in Australia such as AGD66 and AGD84. GDA94 also offers a good level of agreement with other geocentric reference frames. In particular, consistency with WGS84 is currently acceptable for the vast majority of GPS applications and this was a principle objective in moving to GDA94 , thereby facilitating routine use of GPS in Australia. The principle disadvantage of GDA94 is that it is a static datum and therefore cannot accommodate tectonic plate motion. The practical consequence of this limitation is that GDA94 coordinates are drifting away from a true geocentric frame of reference at a rate of about 6 cm per year. Thus GDA94 and WGS84 are becoming less compatible with the passage of time.

Question 1 *(Geodetic Coordinates and Reference Frames)*

(b) Briefly describe the four steps in datum realisation. *(4 marks)*

Datum realisation is the step that follows datum definition. It turns the concept of a datum contained in the design into reality so that the datum can be accessed and used. There are typically four steps in the realisation process :

Monumentation – is the processing of placing points in the ground to which coordinates will be assigned and subsequent survey networks can be linked.

Observation – is the process of connecting the points placed in stage 1 by measurement so as to construct a geodetic network.

Computation – is the process of adjusting the network of measurements (within an existing framework of control – e.g. ANN) so as to obtain coordinates relative to the new datum.

Publication – is the final phase of the datum realisation process whereby coordinates for monumented points are released to the public for use.

Question 2 *(Introduction to GPS)*

(a)	GPS satellites transmit a satellite message consisting of three data blocks. Describe the various contents of each block. *(10 marks)*

The satellite message is a 50 bps data stream transmitted on the L1 and L2 carriers. The satellite message provides users with satellite specific and constellation wide information essential for the reliable computation of position by a GPS receiver. The satellite message is transmitted as a series of *frames*. Each frame is 30 seconds long (1500 bits) and is divided into five sub-frames, each of 6 seconds duration (300 bits).

<u>Datablock 1 (sub-frame 1)</u> relates specifically to the transmitting satellite and is repeated in every frame (i.e. every 30 seconds). Data block 1 contains the following information :

Clock coefficients – the parameters of a second-order polynomial that allow the clock of the transmitting satellite to be corrected to GPS system time (to an accuracy of 7 nano seconds).
Satellite health – a satellite health indicator that informs the user/receiver whether or not the satellite should be used. For example, a satellite will be set unhealthy if it is being maneuvered in its orbit.
URA – the user range accuracy is an indicator of the quality of the satellite C/A-code transmission. This value is less important than it was in the days of Selective Availability when the presence or absence of SA could be determined by the URA.

<u>Datablock 2 (sub-frames 2 and 3)</u> is again specific to the transmitting satellite and included in every frame. Data block 2 contains the 16 parameters of the broadcast ephemeris. These values allow the real-time location of the satellite in 3D space to be determined, which is an essential requirement for real-time positioning.

<u>Datablock 3 (sub-frames 4 and 5)</u> relates to other satellites in the GPS constellation and contains the following information :

Message – space allocated for the transmission of other information to users (rarely used).
Ionospheric parameters – the 8 parameters of the Klobuchar ionospheric model used for coarse correction of the C/A-code pseudo-ranges for the influence of ionospheric refraction.
UTC-GPS – the number of seconds offset between GPS system time and the international time standard (UTC).
Almanac data – approximate orbit information for other satellites in the constellation to allow a receiver to quickly locate a satellite and also for mission planning purposes. The almanac is a truncated ephemeris (less accurate).
Clock parameters – required to correct the clocks of other satellites in the constellation to GPS system time.
Satellite health – health flags for the other satellites.

Finally, it should be noted that with a constellation of 24 satellites, 25 frames are required to transmit the full satellite message (25 * 30 seconds = 12.5 minutes).

Question 3 *(Code Based GPS Positioning)*

(a) Develop the non-linear pseudo-range observation equation, taking care to define each term. *(5 marks)*

By definition, the code pseudo-range is the time difference between the satellite clock at the time of signal transmission (t) and the receiver clock at the time of reception (T). In mathematical form, this is given by :

$$\tau = T - t$$

However, both the satellite clock and the receiver clock will be offset from GPS system time. We denote the satellite clock offset by dt and the receiver clock offset by dT, the GPS system time at the time of transmission as t_{GPS} and at the time of reception as T_{GPS}. Then we can write :

$$t = t_{GPS} - dt$$
$$T = T_{GPS} - dT$$

Thus the pseudo-range observation equation can be re-written as :

$$\begin{aligned}\tau &= (T_{GPS} - dT) - (t_{GPS} - dt) \\ &= (T_{GPS} - t_{GPS}) - (dT - dt) \\ &= \tau_{GPS} - (dT - dt)\end{aligned}$$

Where τ_{GPS} is the true time difference between transmission time and reception time. Multiplying this last equation by the speed of light (c) converts the equation from units of time to units of length, noting that $c\tau_{GPS}$ gives the true geometric range (R) between the receiver and the satellite

$$c\tau = c\tau_{GPS} - cdT + cdt$$
$$\rho = R - cdT + cdt$$

Finally, the geometric range can be expressed as a function of the 3D geocentric cartesian coordinates of the satellite (X^S, Y^S, Z^S) and the receiver (X_R, Y_R, Z_R).

$$\rho = ((X^S - X_R)^2 + (Y^S - Y_R)^2 + (Z^S - Z_R)^2)^{1/2} - cdT + cdt$$

This equation is the non-linear form of the observation equation for the code pseudo-range observable.

Question 3 *(Code Based GPS Positioning)*

(b)	Explain how the various Dilution of Precision (DOP) factors can be derived from the solution of the equation derived for Question 3(a). *(5 marks)*

When four or more satellites are observed, it is possible to compute a least squares solution using the above equation to estimate the receiver coordinates and receiver clock offset. The equation must be linearised and the least squares solution computed to yield estimates for the four parameters. The least squares solution will take the form :

$$\mathbf{m} = \mathbf{A}\mathbf{x} + \mathbf{v}$$

Precision of the least squares estimates will be given by :

$$\mathbf{V}_x = \mathbf{A}^T \mathbf{V}_m^{-1} \mathbf{A}$$

Which can be written as :

$$\mathbf{V}_x = \begin{bmatrix} \sigma_X^2 & \sigma_{XY} & \sigma_{XZ} & \sigma_{XT} \\ \sigma_{YX} & \sigma_Y^2 & \sigma_{YZ} & \sigma_{YT} \\ \sigma_{ZX} & \sigma_{ZY} & \sigma_Z^2 & \sigma_{ZT} \\ \sigma_{TX} & \sigma_{TY} & \sigma_{TZ} & \sigma_T^2 \end{bmatrix}$$

The various DOP factors are all derived from the $\mathbf{V}_x$ matrix. For example :

$\text{PDOP} = (\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2)^{1/2}$ (Position)

$\text{GDOP} = (\sigma_X^2 + \sigma_Y^2 + \sigma_Z^2 + c^2\sigma_T^2)^{1/2}$ (Geometric)

$\text{TDOP} = \sigma_T$ (Time)

HDOP (horizontal DOP) and VDOP (vertical DOP) can also be computed, but the $\mathbf{V}_x$ matrix must be transformed into a local (north, east up) system to obtain these values.

Question 4 *(Miscellaneous)*

(a)	Compare and contrast GPS today with GPS after completion of the planned modernisation program. *(5 marks)*

The main features of GPS modernisation will include :

- Setting of Selective Availability to zero
- New civilian code on L2, to be known as L2C
- New military code on L1 and L2, to be known as M-code
- New civilian frequency, known as L5
- Enhancements to the Control Segment (details not specified)

The benefits of GPS modernisation include :

- Improved accuracy for code based positioning applications
- Improved integrity for critical users (e.g. civil aviation)
- Improved signal availability
- Improved ionospheric modelling
- Improved ambiguity resolution
- Less vulnerability to jamming

Question 4 *(Miscellaneous)*

(b)	Describe the main differences between the broadcast ephemeris and the precise ephemeris. Under what circumstances might a GPS user choose to use the precise ephemeris? *(5 marks)*

The broadcast ephemeris provides a *prediction* of satellite location in real time and is transmitted to users as part of the GPS satellite message. The broadcast ephemeris is provided in the form of 16 parameters of a modified (perturbed) Keplerian orbit. Because it gives predicted satellite orbits, the accuracy of the broadcast ephemeris is limited to about ±2.0 m. The broadcast ephemeris is computed by at the Master Control Station of the GPS Control Segment. The computation takes observational and atmospheric data from the five GPS ground control stations. The solution is computed in two-parts – a 7-day reference ephemeris (6 parameters) and perturbations to the reference ephemeris (10 parameters).

The precise ephemeris on the other hand is available in three forms – ultra-rapid, rapid and final. Precise ephemeredes are obtained by post processing satellite tracking data collected by the IGS (International GNSS Service) network. There are over 200 globally distributed tracking stations in the IGS network that contribute data to the precise ephemeris computations. Only the ultra-rapid IGS orbits are available for real-time positioning, but even these are substantially better (10 cm) than the broadcast ephemeris. The rapid orbits are available 17 hours after the data was collected and offer an accuracy of better than 5 cm. The final orbits take 13 days to compute and again offer accuracy better than 5 cm. IGS precise ephemeris products are available free of charge via the IGS web-site.

Precise ephemeris products would be used in cases where long baselines (100's – 1000's of km) are being observed and high accuracy is absolutely essential. In such cases, the delay in waiting for the precise ephemeris is secondary to achieving the best possible solution.