Department of Geomatics
451-337 Satellite Positioning and Geodesy
Mid-Semester Test 2007

— WORKED SOLUTIONS —

Question 1 *(Geodetic Coordinates and Reference Frames)*

(a) Provide a complete description, including diagrams where appropriate, of the three coordinate systems commonly used in geodesy. *(10 marks)*

The three coordinates systems commonly used in geodesy are the cartesian (X,Y,Z) system, the geographical system (φ, λ, h) and the map grid system (E, N, h). These systems are inter-related so that, given coordinates in one system, coordinates in the other two systems can be derived by using appropriate formulae. Below is a description for each of the three coordinate systems.

Cartesian Coordinates (X,Y,Z)

The following diagram shows both the cartesian and the geographical coordinate systems, with particular reference to a point P.

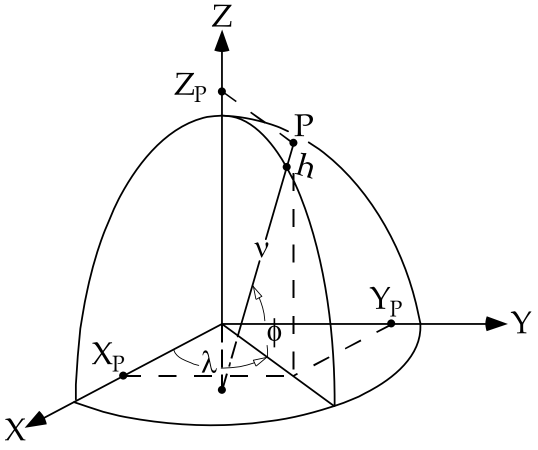

Figure 1 – Cartesian and geographical coordinates

Cartesian coordinates have the origin of the right-handed axis system at the centre of the relevant geodetic ellipsoid. If the ellipsoid is earth-centred, the Cartesian coordinates are said to be *geocentric*. The XZ-plane defines the zero meridian for longitude (which is the meridian of Greenwich in the geocentric case). The Z-axis runs through the CIO (Conventional International Origin) which defines the position of the mean north pole for the period 1900-1905. The XY-plane coincides with the zero parallel for longitude (which is the equator in the geocentric case).

Geographical Coordinates (φ, λ, h)

Geographical coordinates are latitude, longitude and ellipsoidal height and are likewise illustrated in Figure 1.

Latitude (φ) is an angle measured in the meridian plane from the equator (or the zero parallel) to the ellipsoid normal that passes through the point of interest (P in Figure 1).

Latitude is reckoned positive north of the equator and negative to the south. Thus latitude will fall in the range ±90° or 90°S to 90°N.

Longitude (λ) is an angle measured in the equatorial plane from the zero (Greenwich) meridian to the meridian through P. Longitude is reckoned positive to the east of Greenwich and negative to the west. Thus longitude will fall in the range ±180° or 180°E to 180°W.

Ellipsoidal height (h) is the distance (in metres) reckoned along the ellipsoid normal through P between the surface of the ellipsoid and the point P.

Map Grid Coordinates (E, N)

Strictly speaking, map grid coordinates are only 2D, though they are typically coupled with either the ellipsoidal or orthometric height to define the third dimension of position. (E,N) coordinates are derived from a projection of the (φ,λ) coordinates. The most commonly used projection for geodetic purposes is the Universal Transverse Mercator (UTM) projection, which, in Australia is the basis for the MGA coordinate system. UTM is a conformal tranverse cylindrical projection (angles are maintained). For this projection, the earth is divided up into 60 UTM zones, each being 6° wide in longitude. Each zone has a central meridian and a false origin to ensure the derived map grid coordinates are everywhere positive within a zone. The false origins are different for the northern and the southern hemispheres. Scale also varies within each zone due to the distortion introduced by the projection process. On the central meridian the scale factor is 0.9996 and increases to 1.0005 on the extremities of the zone. The scale factor will be 1.0000 where the projection cylinder and the ellipsoid surface are coincident.

Question 2 *(Introduction to GPS)*

(a) Explain and illustrate the flow of data within the Control Segment and between the Control Segment and Space Segment. *(5 marks)*

The control segment consists of three types of station. There are 12 *Monitor Stations* that track GPS satellite data and transmit that data in the form of 15 minute smoothed ranges to the *Master Control Station* (MCS) at Schriver Airforce Base, Colorado Springs. The MCS receives and processes the range data from the Monitor Stations to compute, amongst other things, satellite orbit parameters (16 parameters), satellite clock correction parameters (3 parameters) and ionospheric parameters (8 parameters) for each satellite. These data are sent to three *Ground Control Stations* for uploading to the GPS satellites. This information is supplied to the satellites in the form of the GPS Navigation Message which the GPS satellites subsequently transmit to users for positioning and navigation purposes.

Data flow within the Control Segment is shown in Figure 2.

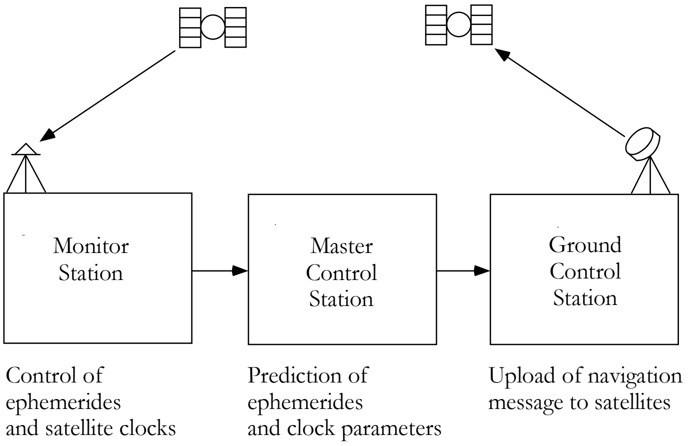

Figure 2 – Data flow within the GPS Control Segment

(b) Describe the measurement signals transmitted by a GPS satellite, including the relationship between them and the benefits, limitations and some possible applications of each. *(5 marks)*

GPS satellites transmit a total of 5 measurement signals that can be used for positioning purposes. All these signals are derived from the on-board satellite oscillator (clock) and are related to its fundamental frequency (10.23 MHz) by a pre-defined multiplier (see Figure 3). The 5 measurement signals are :

- *L1 carrier* – A sinusoidal carrier signal with a wavelength of approximately 19 cm. The L1 carrier provides a precise measurement signal, but these measurements suffers from the need to resolve the number of full wavelengths between the satellite and the receiver (the integer ambiguity). The L1 carrier is primarily used by surveyors for precise positioning applications over short distances (< 20 km). Using only L1, distances must be kept short because of the influence of the ionosphere.

- *L2 carrier* – A sinusoidal carrier signal with a wavelength of approximately 24 cm. Most commonly used in conjunction with the L1 carrier, the L2 carrier then provides a dual frequency measurement system used for high precision positioning over long distances (> 20 km). The combination of L1 and L2 measurements facilitates ionospheric elimination and/or rapid ambiguity resolution.
- *C/A-code (on L1)* – The *Coarse Acquisition* (or Civilian Access) code, which is a pseudorandom noise (PRN) binary sequence with a repitition period of 1 millisecond (300 km) and a wavelength of approximately 300 m. The C/A-code is the basis of the Standard Positioning Service (SPS) which is available to all civilian users of GPS offering a stand-alone accuracy of ±10m. The C/A-code is unique to each satellite allowing for rapid satellite identification and the measurement of unambiguous ranges. The C/A-code is the most basic and most widely used GPS signal and is employed for coarse mapping and recreational purposes.
- *P-code (on L1 and L2)* – the *Precise* (or *Protected*) code is restricted to military users of GPS by means of an encryption strategy (Anti-Spoofing). The P-code is again a PRN binary sequence, but with a repetition period of 267 days. Each satellite is assigned a unique 7 day segment of the P-code. The P-code is the basis of the Precise Positioning Service (PPS) giving authorised users a single receiver positioning accuracy in the order of ±1-2 m. The P-code is primarily used for military purposes such as missile guidance, ship and air navigation and troop coordination on the ground.

The 5 measurement signals and the relationships between them are shown in Figure 3.

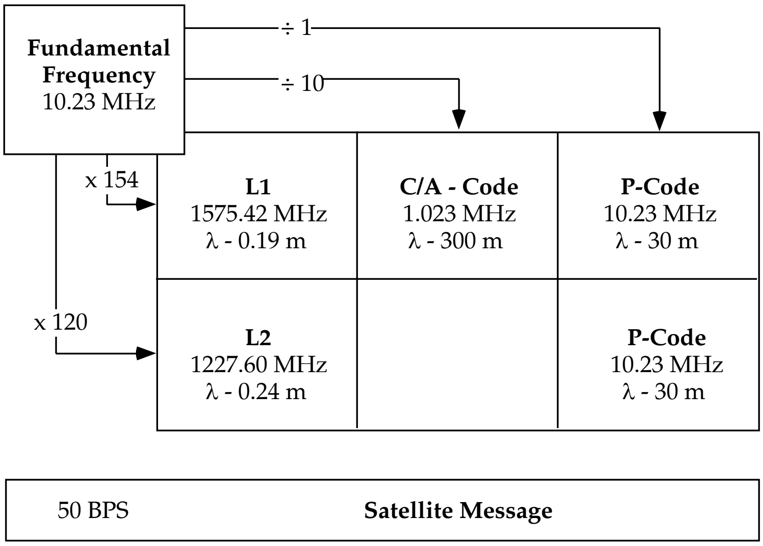

Figure 3 – *The GPS measurement signals*

Question 3 *(Code Based GPS Positioning)*

(a) Explain the principle of DGPS and elaborate on why differential GPS yields significantly better accuracy than single receiver positioning. *(5 marks)*

Differential GPS (DGPS) is based on the principle that measurements taken by *nearby* GPS receivers will be affected by similar errors (due to the spatial correlation between many of the GPS error sources). It follows therefore that determining the position of one receiver (the *rover*) relative to the other (the *base*) will result in the cancellation/minimisation of the influence of the common errors and a subsequent improvement in positioning accuracy. Stand-alone GPS positioning yields an accuracy of ±10 m, whereas the accuracy achievable using DGPS is closer to ±1-2 m. The spatially correlated errors that are minimised by employing the DGPS technique include satellite orbit and clock errors as well as atmospheric errors such as ionospheric and tropospheric refraction.

(b) Define the GPS pseudorange observable and describe the principle upon which a GPS receiver measures a pseudo-range. Explain how such measurements can be combined to determine receiver location. *(5 marks)*

The GPS pseudo-range observable is based on the measurement of the time taken for a C/A-code signal transmitted by a satellite to be received at the antenna of a GPS receiver, as illustrated in Figure 4.

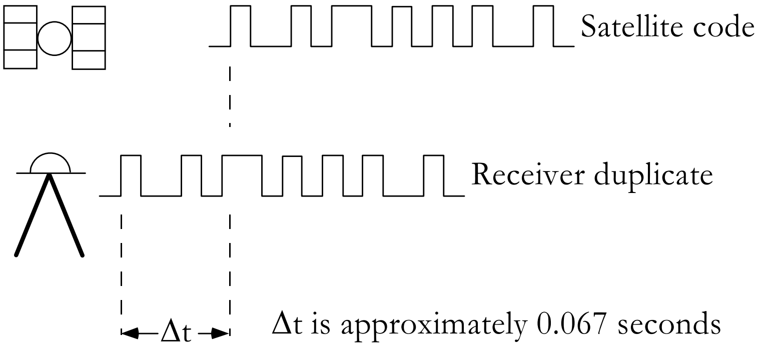

Figure 4 – Measurement of the GPS pseudorange

The Δt term shown in Figure 4 is the pseudorange observable. Since the physical measurement of this quantity is affected by both the satellite clock and the receiver clock and since these clocks are both subject to error, the measured pseudorange is corrupted by these clock errors. It is for this reason that the pseudorange is given its name – it is not a *true* range.

As also shown in Figure 4, the GPS receiver measurea the pseudo-range using a code-correlation technique. This means that the GPS receiver generates a duplicate of the satellite's transmitted code which is subsequently shifted in time to correlate with the incoming satellite signal. This shift equates to the transmission time and thus becomes the pseudorange measurement, when multiplied by the speed of light.

Combining pseudorange measurements to multiple satellites at a single receiver allows the position of a receiver to be determined. This is effectively a resection by distance. The measured pseudoranges are combined in a least squares solution to solve for the receiver coordinates based on the known coordinates of the satellite. It should be noted that this process also requires the receiver clock offset to be resolved and that therefore a minimum of four satellites is required to solve for 3D receiver position. The satellite clock error is dealt with using the clock correction parameters provided as part of the satellite message.

Question 4 *(Miscellaneous)*

(a) What were the main steps taken to define and realise GDA94? *(5 marks)*

The main steps taken in the *definition* of GDA94 were :

- *Size and Shape* – GDA94 is based on the GRS80 ellipsoid which has well established parameters to define its size (a) and shape (f)
- *Location* – The main motivation for the new datum was to move away from a regional to a geocentric frame of reference. As such, the centre of the ellipsoid was adopted to coincide with the earth's centre of mass (at the nominated epoch – January 1994)
- *Orientation* – Being geocentric, the orientation of GDA94 is pre-defined so that the Z-axis of the geocentric Cartesian system passes through the CIO (see Q1), the X-axis falls in the Greenwich meridian and the XY-plane coincides with the equator

The main steps in the realisation phase of GDA94 were :

- *Monumentation* – In addition to using existing geodetic monumentation throughout Australia, new sites were established for the Australian Fiducial Network (AFN), consisting of concrete pillars and permanently installed GPS receivers
- *Measurement* – An extensive program of measurement was carried out in two major phases. Firstly, the AFN sites were connected to IGS sites to establish a linkage to the ITRF. Secondly the AFN sites were connected to the ANN sites by a campaign of GPS observations. Subsequently, existing geodetic observations were used to bring GDA94 down to lower layers of the geodetic and survey control network
- *Computation* – The computation process was also carried out in stages. Firstly ITRF92@1994.0 coordinates were calculated for the AFN sites through their connection to the IGS network. These were subsequently used for control in the re-adjustment of lower layers of the control networks using the observations described above to propagate GDA94 coordinates through the ANN And then throughout Australia
- *Publication* – Upon completion of the adjustments, surveying, mapping and geodetic agencies throughout Australia began to publish GDA94 coordinates for all survey control marks from about January 2000.

Question 4 *(Miscellaneous)*

(b) Give a complete description of the 3D conformal transformation model used in geodesy and provide some examples of its use. *(5 marks)*

The 3D conformal transformation model used in geodesy is a 7 parameter transformation that maintains shape (hence *conformal*). The 7 parameters comprise:

- 3 orthogonal shifts (ΔX, ΔY, ΔZ)
- 3 rotations (RX, RY, RZ) and
- a scale factor (λ)

The model takes Cartesian coordinates in one frame of reference (x,y,z). These coordinates are rotated, scaled and shifted to obtain coordinates in the second frame of reference (X,Y,Z). Mathematically, the conformal transformation can be represented by the vector equation:

$$\mathbf{X} = \mathbf{\Delta} + \lambda\mathbf{R}\mathbf{x}$$

Diagrammatically, the relationship is shown in Figure 5.

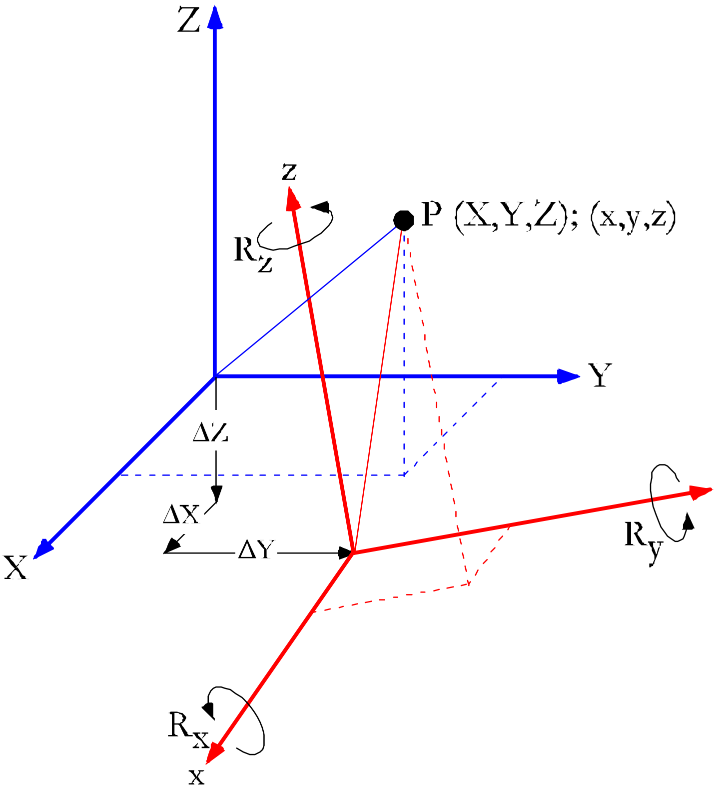

Figure 5 – The 7 parameter conformal transformation

The most common application of the conformal transformation model is to transform Cartesian coordinates from one frame of reference to another. For example it is widely used to move coordinate data from old geodetic datums such as AGD66/84 to the newer GDA94.