Towards a Probabilistic Time Geography

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ABSTRACT

Time geography uses *space-time volumes* to represent the possible locations of a mobile agent over time in a *x-y-t* space. Space-time volumes are qualitative statements, enabling qualitative analysis. In this paper these statements will be quantified, modeling an agent's possible locations from a stochastic perspective. With such a model time geography can be improved in expressiveness as well as accuracy.

Categories and Subject Descriptors

G.3 [**Probability and statistics**]: Distribution functions, time series analysis; H.1.1 [**Models and principles**]: Systems and information theory—*information theory*; I.6.8 [**Simulation and modeling**]: Types of simulation—*Discrete event simulation*

General Terms

Time geography

1. INTRODUCTION

Tracking systems log sequences of discrete locations of mobile agents, technically because observations are always discrete, but more practically due to limited bandwidth or database limitations. Locations of the mobile agents in between two consecutive logged locations are frequently modeled by interpolation [6]. Interpolation leaves some uncertainty about the location of agents between the logged locations, depending on degree of freedom of the movement and the time interval between two observations. A more rigorous approach to model the possible locations of mobile agents is provided by time geography [7, 11, 19, 12], representing and analyzing all possible locations of an agent in space and time in form of *space-time volumes*. The most basic of these volumes, the *space-time cone*, is studied here. It describes the potential locations of an agent if its location is known only at one time. Time geography analyzes such discrete geometries.

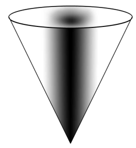

Figure 1: The probabilistic space-time cone.

Interesting operations in time geography are intersections of these volumetric geometries. By this way time geography facilitates qualitative statements such as about potential locations at a particular time, and potential encounters of agents. A quantification of the results was, to my knowledge, never tried. Quantifications based on the relative size of the intersections would be misleading, as it ignores the likelihoods of finding the agent at particular locations at particular times within the volume. These likelihoods do not follow an equal probability distribution. The probability of finding an agent somewhere at a particular time t_i depends on factors such as their goal-orientation or the regularity of their behavior. But even a completely undirected random movement does not lead to an equal probability distribution. Thus, the hypothesis of this paper is: A probability distribution in space-time volumes is non-equal and can be determined from *a priori* knowledge about the agent's behavior. Knowing a probability distribution will facilitate quantitative analysis in time geography.

To prove this hypothesis, this paper concentrates on probabilistic space-time cones, leaving an extension for space-time prisms for future work. Drawing on a discrete *space-time aquarium* [5], a probabilistic space-time cone will be introduced (Fig. 1). Without loss of generality a *discrete* probabilistic space-time cone approximates the continuous probability distribution of the location of a mobile agent by an appropriate resolution. Within this discrete volumetric model the probability distribution will be derived from convolution.

Questions involving a quantification were not suggested so far, except a not further developed suggestion of fuzzy space-time volumes [13]. These questions would be of the kind: What is the most probable arrival time of an agent A at a particular location C? Or what is the probability that two agents A and B have met by t_i? These questions are relevant for applications such as search in rescue and criminology, collision avoidance, or the estimation of arrival time of individual agents.

2. BACKGROUND

Probabilistic time geography touches several areas, among them (classical) time geography and random walks.

Time geography. Assume that an agent can move in any direction and is limited only by a maximum speed $v_{\max}$, then a representation of all reachable locations of this agent is the *space-time cone*, a right cone in isotropic *x*-*y*-*t*-space [7]. The cone apex represents the agent's location at t_0, and the aperture the maximum speed of the agent, such that a cone base represents the set of locations the agent may settle at a time $t_i > t_0$ (Figure 2a).If the agent's location is known at t_0 and then again at t_n, the possibly reachable volume is described by a *space-time prism* (sometimes called *bead* [8]), which is straight if the agent returns at t_n to the location of the origin at t_0 (Figure 2b); otherwise it is oblique (Figure 2c). The *space-time path* is a degenerated space-time prism in form of a linear trajectory, and a stationary space-time path is called a *space-time station* (Figure 2e). A parallel projection of a space-time volume on the *x*-*y*-plane describes all places a moving agent can possibly have reached and is called their *potential path area.*

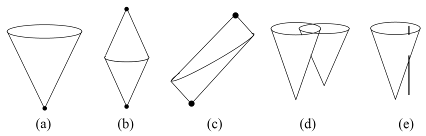

Figure 2: Traditional space-time volumes.

Space-time volumes facilitate qualitative analysis. Given two agents A and B time geography is interested, for example, whether by t_i agent A has possibly met agent B (Figure 2d), or whether agent A has possibly reached location C (Figure 2e). Such qualitative questions can be answered by testing whether the intersection of their corresponding space-time volumes is empty or not. The cone base at t_i represents the uncertainty about an agent's location at t_i, hence, for the analysis at t_i one can apply the usual relational calculi for extended objects [4, 17]. Even extensions of these calculi for uncertain objects exist [21, 3, 2, 14]. However, the qualitative statements derived from these calculi have to consider the point-like nature of the agents, mapping onto qualitative likelihoods of meetings (*impossible*, *possible*).

A first attempt to allow for temporal uncertainty—i.e., uncertainty on departure or arrival time—or spatial uncertainty—i.e., uncertainty on departure or arrival location—was made by Neutens *et al.* [14], introducing rough space-time prisms. In this model based on rough set theory [15], uncertainty defines a lower volume $\underline{V}$ and upper volume $\overline{V}$ such that $\underline{V}$ is describing the space-time volume certainly accessible, and $\Delta V = \overline{V} - \underline{V}$ the volume possibly accessible. While this model is still qualitative, with a three-valued logic (true, false, maybe), its relationship to the present paper is interesting: while Neutens *et al.* consider the uncertainty of departure or arrival (time and location), here the uncertainty about the agent's movement decisions is modeled.

Random walks. The basic argument in this paper is that unknown movements of agents can be modeled as random walks.Unbiased random walks approximate continuous diffusion processes such as Brownian motion, i.e., non-goal-directed behavior. Rational agents most of the time move towards goals, i.e., their movements are biased. While this paper starts out to design a probabilistic time geography with unbiased random movements, further work must extend the model to biased random walks. A bias can easily brought in a random walk by biased transition probabilities, say, towards the destination of a space-time prism.

The behavior of large numbers of mobile agents are frequently modeled by random walks. For example, random walks can be used to simulate large numbers of mobile agents to study higher order patterns such as flocking behavior, crowding, queuing or congestions of agents [9, 1, 10]. In contrast to studying large numbers of agents, the present paper simulates by random walks the many wayfinding options a single agent has. Large numbers of random walk simulations provide frequencies of visited places at particular times, which can be normalized to probabilities.

Random walks are treated in this paper in discrete space and time. Polya has shown for infinite random walks in one- and two-dimensional discrete space that the likelihood of an agent returning to the origin (or, equivalently, reaching any other particular point in space) is one [16]. In contrast, this paper is interested in finite time spans only, as they occur in time geography, and hence, we will get probabilities of an agent reaching a particular location that are smaller than one.

3. UNCERTAINTY OF LOCATION

This section will introduce a random walk in a discrete space-time aquarium, and then develop one approach of computing probabilistic space-time cones from random walks.

Assume an isotropic discrete space-time aquarium that is regularly partitioned into voxels of (Δx, Δy, Δt). Assume an agent moving in this aquarium, starting at t_0 from a known location, and ending at t_n after n time steps of *a priori* unknown movements. The discrete steps of the agent are limited by its maximum speed $v_{\max}$. By choosing an appropriate Δt we can realize $v_{\max} = 1$, and by assuming 4-neighborhood (von Neumann neighborhood) agents can stay where they are, or move one step South, North, West, or East (Fig. 3). Then a random walk consists of a sequence of a move according to the actual heading, and a random assignment of a new heading. This random assignment should be unbiased because no prior knowledge of the agent's goal, preferences or behavior exists.

Large numbers of random walks provide the probability distribution of the location of an agent at any time $t_0 \leq t_i \leq t_n$. From the several possible computations of the probability distribution—by simulation, by combinatorics, by convolution, or by experimental observation—, one approach is developed here for illustration. The theoretic result is a bivariate multinomial distribution with $k = 5$ in the discrete aquarium, or a bivariate normal distribution in a continuous aquarium.

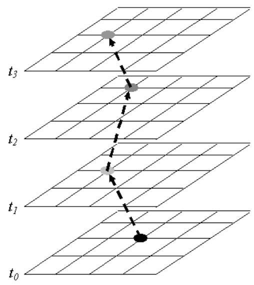

Figure 3: A random walk of $v_{\max} = 1$.

Convolution, or discrete linear spatial filtering of a field a by a kernel h, can be expressed by:

$$b_{x,y} = a_{x,y} \otimes h = \sum_{i=-\infty}^{\infty} \sum_{j=-\infty}^{\infty} h_{i,j}\, a_{x-i,y-j} \tag{1}$$

with b the resulting filtered spatial field. In this most general expression, the neighborhood of a focal point (x, y) is unlimited $(-\infty \ldots \infty)$, but in practical applications its area of influence can be limited to a small neighborhood, supported by Tobler's law: "Everything is related to everything else, but near things are more related than distant things." [20, p. 236]. A typical size of a kernel is only 3×3, describing the neighborhood from $(x - 1, y - 1)$ to $(x + 1, y + 1)$. This kernel has a radius of $d = 1$. In the present context the neighborhood is even physically limited to $d = 1$, by the maximum speed $v_{\max}$ of the agent.

In contrast to typical signal processing, the convolution has to be computed here only locally, in the neighborhood of the moving agent [18]. The 3×3 convolution kernel h reflecting the above described discrete random walk is $h = \{(0, 1, 0), (1, 1, 1), (0, 1, 0)\}$. Applied recursively on a unit impulse function at (x_0, y_0) (the agent's location at t_0) one gets directly frequencies of visits. Using instead a normalized kernel $h' = 0.2 \cdot h$ provides probabilities directly. For example, the probability for an agent to stay between t_0 and t_1 in (x_0, y_0) is 0.2. Figure 4 shows the first two convolution steps, where the intensity in grey is proportional to the probability value.

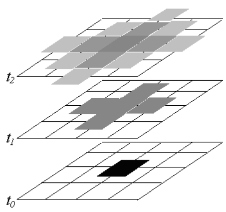

Figure 4: The probability distribution of locations of a mobile agent at sequential time steps.

A recursive formulation of the convolution further avoids the computation of values for empty cells. Thus, for an agent being located at (x_0, y_0) at t_0, the space-time cone between t_0 and t_n consists of $n + 1$ layers k:

$$c_{x,y,k} = \begin{cases} 0 & \text{if } |x - x_0| > k \vee |y - y_0| > k \\ c_{x,y,k-1} \otimes h & \text{else} \end{cases} \tag{2}$$

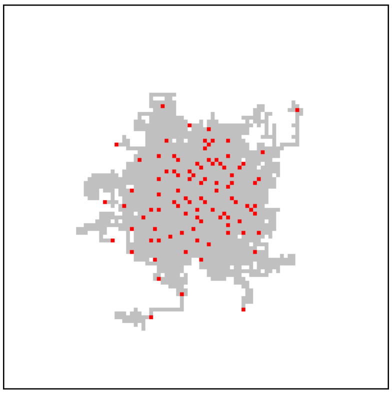

Figure 5: 100 random walks.

This means computational complexity is a function of time only, more precisely, of the n time intervals for which a cone is computed. For each time step i the convolution computes $(2i + 1)^2$ new values ($\mathcal{O}(n^2)$).

4. EXPERIMENTAL EVIDENCE

The theoretically derived probability distributions should show in collected travel data. For compatibility with the assumptions of an unbiased random walk model one needs data of multiple agents traveling between two locations that are traveling independent from each other. For reasons discussed above, collected data of an individual would not suffice due to their rather regular movement patterns. Such data sets exist.

Synthetic data can be produced by the model described above. For example, Figure 5 presents the base of a space-time cone created by 100 concurrent random walkers (black spots) over 200 iterations, who all started in the apex of the cone. The total explored area is shown in grey. The figure was computed by a Swarm simulation provided online by the Complexity Virtual Lab, Monash University. This tool allows to set a probability of the agents changing their directions. This probability was set here to 80%.

Note that the radius of the cone base after 200 iterations is 200, and is much larger than the total explored area in Figure 5. This means it is unlikely that an agent reaches outer areas of the base. Even in the explored area the density of agents varies, with the highest probability to find an agent in the center.

5. DISCUSSION AND CONCLUSIONS

This paper has presented first steps into a probabilistic time geography, or more generally into a quantitative time geography. It addresses questions such as how likely it is to find an agent in a particular area, or how likely it is that two agents meet. The paper has clearly shown that the probability distribution of finding an agent within a space-time volume is non-equal. This is an important result, because it disqualifies any attempt to quantify by relative sizes of intersections.

The paper also discusses the importance of sufficient *a priori* knowledge about the agent's behavior. Different behavior forms significantly different probability distributions. For this paper, an agent's movement behavior was assumed to be unbiased. Unbiased movement can be assumed for example for an average behavior from a large number of independently moving agents, but also for a strolling agent without a particular goal, or perhaps even for an agent with goal-directed movements over longer time spans. Unbiased movement is well captured by the *a priori* knowledge expressed by the parameters of classic space-time cones: known is only the location of an agent at a time t_0, and their maximal speed $v_{\max}$, and nothing else.

In general, however, goal-directed behavior violates these assumptions, and hence, goal-directed movements do need adapted probability distributions. If the modeler knows about the goals, this additional *a priori* knowledge can be brought in by biasing the transition probabilities and changing by that way the probability distribution. These extensions are mentioned, but their development are beyond the scope of the paper.

The presented model lays the foundations for a large number of future questions. Among these are a further refinement of the probabilistic cone model for different types of environments or different agent behaviors, the completion of the cone to a probabilistic space-time prism, and the further development of reasoning mechanisms with these probabilistic space-time volumes.

Acknowledgements

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